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Supersonic Wave/Blade-Row Interactions Establish Boundary Conditions for Unsteady Inlet Flows

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THE computation of unsteady flows in high-speed airbreathing inlets requires a compressor-face boundary condition (CFBC). The physical basis for the analytical or numerical formulation of a realistic CFBC is the acoustic reflection coefficient of the operating compressor. In Ref. 1 an approximate, one-dimensional method is offered to calculate this information for a single blade-row compressor, when the total approach velocity (vector sum of axial and tangential components, relative to the rotor blades) is subsonic. The present Note extends the calculation to the practically important case where the total approach velocity is supersonic and the axial component is subsonic. (In this Note, the term "supersonic" refers to this limited range.) The background, motivation, physical model, method of analysis, and nomenclature described in Ref. 1 apply to this Note without change and are not repeated here. Familiarity with Ref. 1 is essential to the understanding of this Note.

In supersonic cascade flows expansion and compression waves can propagate upstream and can modify the approach flow, which makes them considerably different from subsonic cases. It has been well established[2,3] that a steady, uniform, supersonic flow upstream of an infinite, linear cascade of blades can exist only for a unique incidence angle. The existence of such an incidence implies that an upstream moving acoustic disturbance (which might be initiated by a downstream-moving disturbance arriving to the blade row) cancels the initial disturbance, restoring the undisturbed upstream flow. The unique incidence angle can be determined in the knowledge of the blade geometry and the upstream Mach number (methods given in Ref. 2). The unique incidence angle is generally small, several degrees only.

In the following, the exit Mach number is assumed to be subsonic, which is a practically common situation. This assumption also defines a unique steady flow when using the simple mean flow method of Ref. 1. The choice of subsonic outflow implies the presence of shocks (and, hence, a total pressure loss) in the blade passage. The effect of total pressure losses on the reflection coefficient is demonstrably small and a reasonable estimate is sufficient. Shock loss may be estimated as that associated with a normal shock at the upstream Mach number M_u. The estimation of viscous losses may be made on the basis of empirical information valid for similar blade geometries.

The analysis deals with a transient initiated by the arrival of an acoustic step change to the blade row, the goal being the prediction of the magnitude of the reflected wave, which is also a step change. The equations used in the present analysis are the same as those of Ref. 1, with one exception. In the subsonic case, it was assumed that the direction of the exit flow, after completion of the transient, is the same as the direction of the undisturbed exit flow [Eq. (25) in Ref. 1]. In the supersonic case, this requirement is dropped, and the unique incidence requirement is imposed on the entering flow. It is assumed that the response of the blade row has to be such as to restore the incidence angle (in region 3) to its original, unique value (in region 1). This assumption implies that

$$w'_{3y} = w'_{3x} \tan \beta_u \tag{1}$$

Because neither the incident nor the reflected acoustic wave can change the tangential velocity, the tangential velocity disturbances are zero in all upstream regions (1, 2 and 3). Equation (1) then forces the axial disturbance in region 3 to be zero also, which means that the region 3 velocity vector is identical to the undisturbed, region 1 velocity vector. The conclusion is that the imposition of Eq. (1) is equivalent to the assumption of constant (upstream) velocity. This is one of the traditional assumptions that has been applied in the past, without justification, to flows with any approach Mach number. The analysis of Ref. 1 showed that this assumption does not apply for a subsonic approach flow. The present analysis shows that the constant velocity condition is in fact applicable in the supersonic range.

The calculations are simple and are omitted. The acoustic wave coefficients are as follows:

$$\boldsymbol{A}_- \equiv \frac{p'_3 - p'_2}{p'_2} = 1 \tag{2}$$

$$\boldsymbol{A}_+ \equiv \frac{p'_4}{p'_2} = \frac{2M_{ux}\sqrt{\chi}}{\sigma(1 + M_{dx})} \tag{3}$$

The analysis also predicts a downstream-convecting vorticity wave. The corresponding vorticity induction coefficient is given by

$$\boldsymbol{V} \equiv \frac{\rho_u a_u w'_{4y}}{p'_2} = 2\frac{1 - \sqrt{\chi} M_{ux}/\lambda\sigma}{\sqrt{\chi} M_{dx} \tan \beta_d} \tag{4}$$

Equation (2) states that for supersonic flow the reflection coefficient is constant with a value of unity. This result is in excellent agreement with two-dimensional, inviscid Euler solutions obtained by Paynter to the same problem.[4] His solutions clearly indicate that for supersonic approach flows the reflection is such that the velocity direction in region 3 is the same as in region 1, in agreement with Eq. (1). Because $\boldsymbol{A}_-$ is independent of the mean flow properties, the mean flow parameters need not be known to compute this coefficient. In contrast, the calculation of the acoustic transmission and the vorticity induction coefficients does require the knowledge of all steady flow parameters, such as inlet Mach number, inlet/exit flow angles, passage height change, and loss coefficient.

The wave/blade interaction problems for stationary and moving blade rows are identical provided they are stated in coordinates fixed to the blades. If the inflow parameters are given in the stationary frame, then the solution process for a moving blade row consists of transforming all mean flow quantities to the blade-fixed coordinate system and subsequently calculating the wave coefficients using Eqs. (2–4).

For the special case of unloaded flat plates with no area change ($\beta_u = \beta_d = \beta$, $\chi = \lambda = \sigma = 1$, and $M_u = M_d = M$), the subsonic and transonic formulas both predict the same wave coefficients when $M = 1$, that is, the switch from the subsonic to supersonic behavior is continuous and occurs at $M = 1$. Figure 1 shows the variation of the reflection coefficient. Subsonic relations[1] are illustrated using thin lines in the plot whereas supersonic values are shown as thick lines. It is evident that, for subsonic flow, increasing Mach number increases the reflection amplitude. As M_{ux} is increased while keeping β_u constant, M_u becomes equal to one when $M_{ux} = \cos \beta_u$. For axial Mach numbers above this limit, the reflection coefficient is uniformly one for any value of β_u, as given by Eq. (2).

In the general case (curved blades, blade height change, nonzero total pressure loss), the switch from subsonic to supersonic regime involves a discontinuous change at $M_u = 1$ in the value of all three wave coefficients. Figure 2 shows the jump in transmission coefficients, for a representative set of mean flow parameters. Depending on the combination of parameters, the jumps can be significantly higher than those shown in Fig. 2.

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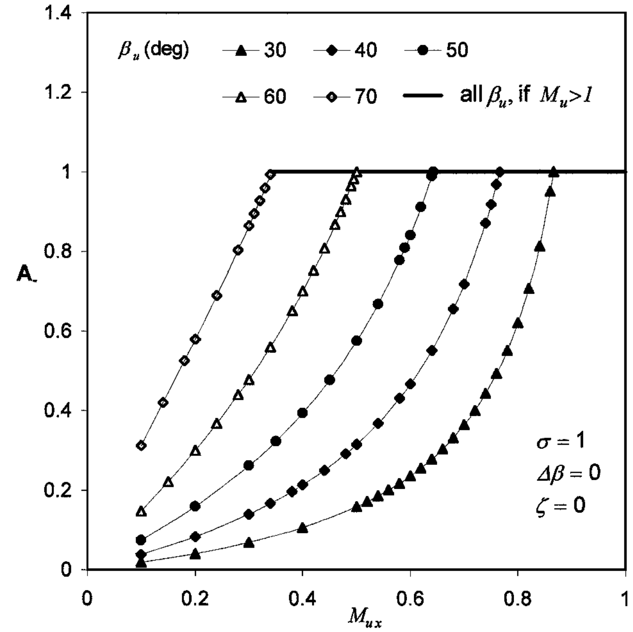

Fig. 1 Acoustic reflection coefficient vs upstream axial Mach number, with inflow angle as parameter, for a flat plate cascade at zero incidence.

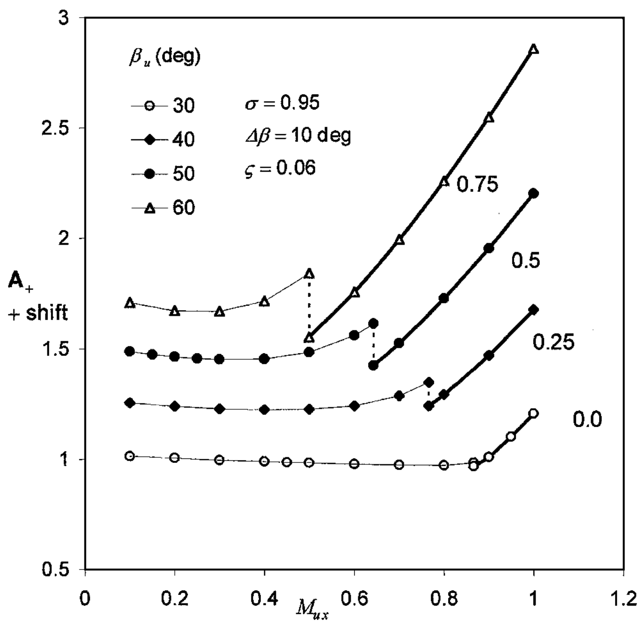

Fig. 2 Acoustic transmission coefficient vs upstream axial Mach number, with upstream flow angle as parameter, for a selected set of parameters. For better readability, curves are shifted upward by labeled amounts.

In Figs. 1 and 2, each point represents a separate realization of a geometry and flow parameter combination, that is, they are design charts in which the geometry is considered to be freely variable. It is also of interest to investigate how the reflection coefficient varies when the operational conditions of a given compressor are changed. Superimposing lines of constant wave coefficient on a standard compressor performance map (total pressure ratio vs corrected mass flow) is a convenient method for the presentation of such off-design behavior. Reflection coefficients were calculated for the example of a transonic research rotor, for which detailed geometry and upstream/downstream flowfield data are available.[5] The maximum measured upstream Mach number was 1.416, and the downstream Mach numbers measured in the blade frame were always subsonic. The performance of the rotor was computed using the time-mean flow method of Ref. 1, using data taken at the mean

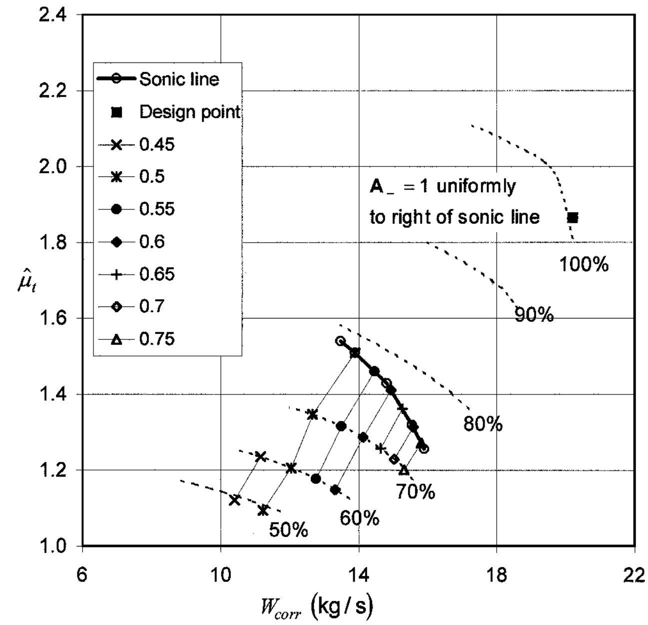

Fig. 3 Contours of constant reflection coefficients overlaid on a compressor performance map; along the sonic line M_u = 1: dashed curves are N_{corr} = const lines.

radius and allowing the incidence angle to vary over the experimentally determined range. Loss coefficients and deviation angles were interpolated from the measured data. Figure 3 shows the results superimposed on the performance map. The map includes a subsonic and a supersonic region, separated by a "sonic line" along which $M_u = 1.0$. In the subsonic region, the reflection coefficient is ranging from 0.4 to 0.75. Right of the sonic line, the approach flow is supersonic, and the reflection coefficient is uniformly equal to unity. The abruptness of the subsonic/supersonic transition is the result of the one-dimensional nature of the theory. If the blades are twisted, and/or the hub/tip ratio is small, then the reflection coefficients predicted for the hub and tip could differ considerably. It is beyond the power of the present one-dimensional theory to predict the resulting complicated three-dimensional effects, but it is likely that real compressors display a gradual change in their reflective behavior as the approach Mach number changes from subsonic to supersonic.

There are no directly comparable experimental data available' primarily because real compressors typically include at least two blade rows. The development of a method to predict reflection coefficients for multistage compressors is clearly desirable.

The implementation of reflection coefficients as boundary conditions into a computer code is a separate and significant challenge that is beyond the scope of this Note. One method, proposed originally by Paynter,[4] has been incorporated into the widely used WIND code by Slater[6] and is available as an official WIND release.

Conclusions

The analytical formulas presented here and in Ref. 1 are intended to replace the ad hoc boundary conditions traditionally used in the computation of unsteady inlet flows, hopefully leading to improved reliability in the assessment of inlet stability and in control system design. Space limitations preclude detailed demonstrations of trends, but an extensive exploration showed that the reflection coefficients can vary widely, from low negative numbers up to unity, depending on the combination of parameters characterizing the situation. The wide range of possible values strongly suggests that the use of proper CFBCs is indispensable for the accurate prediction of unsteady inlet flows.

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